

**Static Analysis and Free Vibration Analysis of Laminated Composite Plate with  
Various Conditions of Arbitrary Limits**Yogesh Mishra<sup>1</sup>, Dr. P.K. Sharma<sup>2</sup>, Deepanshu Bhatt<sup>3</sup>

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**Abstract** — A typical system is a composite material of two materials or composition of several materials (mixed and bonded) on a microscopic scale. Fiber reinforced plastic (FRP) composite laminate plate circular cut through the geometric center of the plate subjected to a uniform load of the transverse pressure is based finite element method. This paper is described Static and free Vibration analysis of laminated composite plate with Various Conditions of Arbitrary Limits. The problem is modeled in the ANSYS finite element software.

**Keywords-** Static and free Vibration, Circular cut-out, boundary condition, ANSYS,

**I. INTRODUCTION**

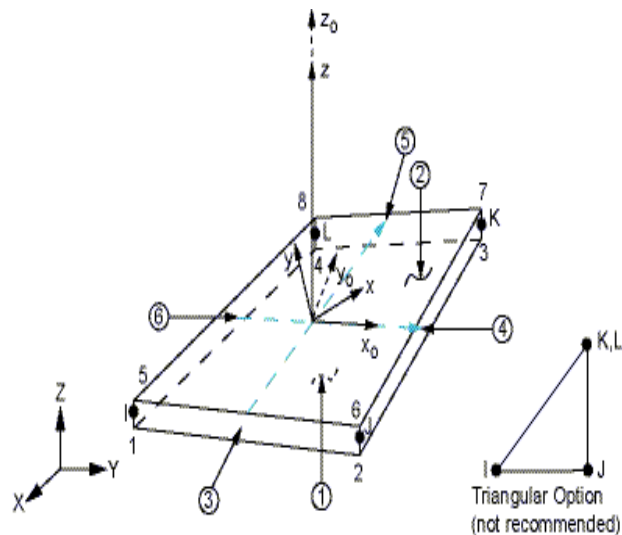
Composite materials have been used in structures for a long period. Structures composed of composite materials are rapidly replacing metal in various disciplines of engineering applications. Plate structures are considered major engineering structures, especially where weight is a major problem. A composite is a structural material that consists of two or more constituents that are combined at a macroscopic level which are insoluble in each other and differ in form or chemical compositions. There are two phases of composite exists namely, reinforcing phase and matrix phase. The reinforcing phase material may be in the form of fibers, particles or flakes and the matrix phase materials are generally continuous such as polymer, metal, ceramic and carbon [1]. The property of composite material depends largely on the properties of the components, the geometry and distribution of phases. The distribution of the armature determines the homogeneity or uniformity of the material system. The strength of the fiber-reinforced composite material increases in the direction of the fibers due to the nature of the continuity of the fiber [2].

Some of the composites are available in natural nature say the timber, where the lignin matrix is reinforced with cellulose fibers and bone, wherein the bone plates of calcium salt and phosphate ions strengthen soft collagen. Composite systems are of reinforced concrete and steel reinforced epoxy graphite fiber / carbon / boron etc. Similarly, the properties and behavior of composite materials are discussed by the authors [3]. Reddy *et al.* [4] developed a higher-order shear deformation theory by taking into account parabolic distribution of the transverse shear strains through the thickness of laminated composite plates, and obtained exact closed-form solutions of symmetric cross-ply laminates and the results are compared with 3-D elasticity solutions and first-order shear deformation theory solutions. A  $C^0$  continuous displacement finite element formulation of laminated composite plates under transverse loads is presented by Pandya and Kant [5] using the HSDT. Cheung *et al.* [6] developed in three linear dimensions, a solution of low deformation for free vibration of rectangular plates, thick layers with different boundary conditions in a process using finite layer. Dhanunjaya K. *et al.* [7] model analysis of thin frp skew symmetric angle-ply laminate with circular cut-out. Reddy [8] studied the free vibration of anti-symmetric angle ply laminated plates including transverse shear deformation using FEM. Reddy and Kuppusamy [9] reported 3-D elasticity solutions natural vibrations of laminated anisotropic plates. Nonlinear static behavior of fibre reinforced plastic (FRP) laminates with circular cutout on the effect of thickness ratio and skew angle are analyzed by Raju *et al.* [10]. Zhang and Kim [11] presented geometrically nonlinear static responses of laminated composite plates by using two new displacement based quadrilateral plate (RDKQ-NL20 and RDKQ-NL24) elements. Kumar and Shrivastava [12] presented free vibration of square laminates with delamination around a central cut out using HSDT. Free vibration of anti-symmetric angle ply laminated plates including various boundary conditions by Khdeir A.A [13]. From the above literatures, it can be seen that many studies has been reported on static and free vibration analysis of laminated Composite plate with arbitrary boundary condition. It is also observed that some of the studies have been done with the help of commercial finite element (FE) tool like ANSYS. It is understood that the numerical study using commercial FE tool not only easy to model but also less time expensive. Hence, the objective of the present work has been stated in following paragraph based on the above observation.

**II. FINITE ELEMENT MODLLING**

The geometry and the finite element model of the layered composite plate. Composite laminate shell element generally SHELL181 ANSYS is used to mesh the geometry of the problem. This item is suitable for modeling of medium thickness thin shell composite laminates. The element is composed of several layers of orthotropic material perfectly glued. The element is quadratic and has six degrees of freedom per node namely, translation in x, y and z directions

respectively, and rotations about in x, y and z axes respectively. This element gives results of high accuracy and discretization involves fewer elements.



**Figure: - 1. Shell181 Element Geometry.**

### III. VALIDITY OF THE PRESENT ANALYSIS

Three validation studies are conducted to test the accuracy of the computer program. For this, three different plates, and the results are compared with those which are available in the literature.

#### 1. Static analysis without cutout:-

A clamped four layers symmetric cross ply ( $0^{\circ}/90^{\circ}$ )s square plate with length 304.8 mm and thickness 24.384 mm under uniformly distributed transverse load is considered as in the reference.

*The material properties as Table 1.*

| Material [10] | $E_1$  | $E_2$  | $E_3$  | $\nu_{12}$ | $\nu_{23}$ | $\nu_{13}$ | $G_{12}$ | $G_{23}$ | $G_{13}$ |
|---------------|--------|--------|--------|------------|------------|------------|----------|----------|----------|
| Value         | 12.605 | 12.628 | 12.628 | 0.23949    | 0.23949    | 0.23949    | 2.154    | 2.154    | 2.154    |

The units of  $E_1$ ,  $E_2$ ,  $E_3$ ,  $G_{12}$ ,  $G_{23}$  and  $G_{13}$  of material are in GPa and support condition are taken as  $u = v = w = \theta_x = \theta_y = \theta_z = 0$ , at  $x = 0, a$  and  $y = 0, b$ .

For the numerical analysis, the central deflection obtained using the present model with different mesh sizes are plotted in Fig.2 and compared with reference [10]. It can be seen easily that the results are showing good agreement with the reference with a very small difference around 2.8%. The present results are showing little bit lower side as compared to the reference.

| Load (KPa)                 | 13.79 | 11.032 | 8.27 | 5.52 | 2.76 |
|----------------------------|-------|--------|------|------|------|
| Zhang Y.X. and Kim K.S[10] | 10.69 | 8.32   | 6.24 | 4.24 | 2.14 |
| Present                    | 10.40 | 8.32   | 6.24 | 4.16 | 2.08 |

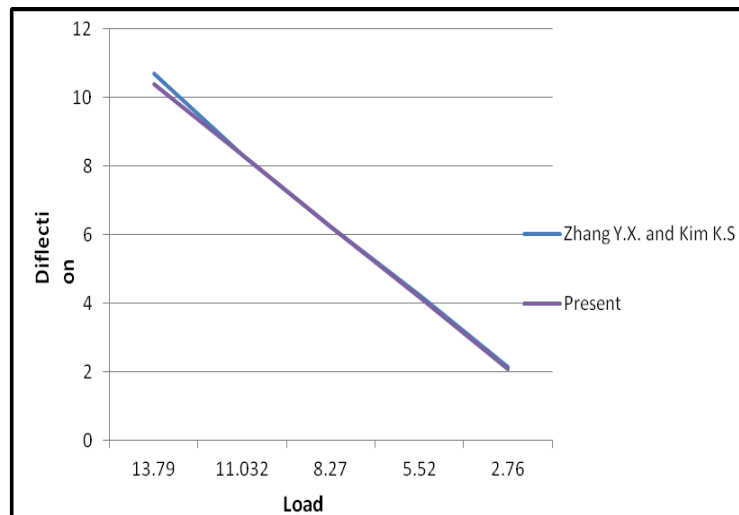


Figure: - 2. Comparison of Central Deflection (in mm) for Cross-ply symmetric laminate

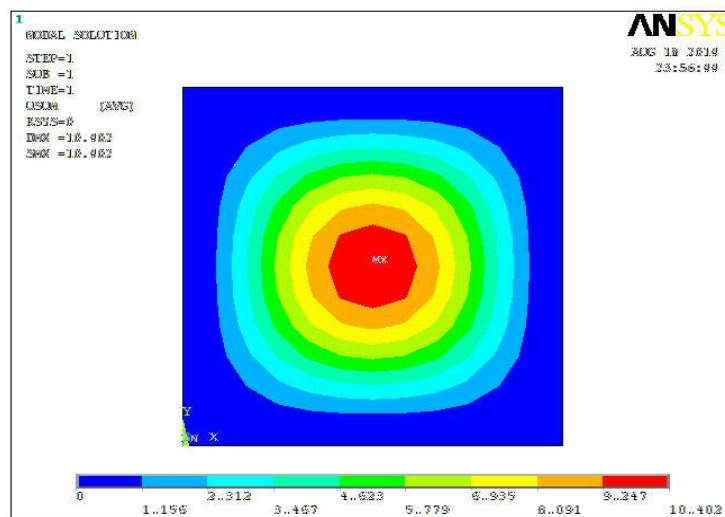


Figure: - 3. Central Deflection for cross-ply symmetric laminate

**2. Static analysis with cutout:-**

A clamped four layers symmetric cross ply ( $0^{\circ}/90^{\circ}$ ) s square plate with circular cutout at centre with side length of 20 mm and the total thickness is 0.5 mm. The transverse pressure is applied on top surface and size of circular cutout is taken as the ratio of diameter to side length is 0.2. To compare the accuracy of the present developed model, numerical results are evaluated based.

The material properties as Table 2.

| Material Constant | $E_1$  | $E_2$  | $E_3$  | $\nu_{12}$ | $\nu_{23}$ | $\nu_{13}$ | $G_{12}$ | $G_{23}$ | $G_{13}$ |
|-------------------|--------|--------|--------|------------|------------|------------|----------|----------|----------|
| Value             | 141.68 | 12.384 | 12.384 | 0.25772    | 0.42057    | 0.25772    | 3.88     | 4.36     | 3.88     |

Table1. The units of  $E_1$ ,  $E_2$ ,  $E_3$ ,  $G_{12}$ ,  $G_{23}$  and  $G_{13}$  of material 1 and 2 are in GPa and the support is  $u = v = w = \theta_x = \theta_y = \theta_z = 0$ , at  $x = 0, a$  and  $y = 0, b$ .

| Load (MPa)                        | 0.5      | 0.4      | 0.3      |
|-----------------------------------|----------|----------|----------|
| Raju V. V., Krishna Murthy V [25] | 0.183 mm | 0.165 mm | 0.107 mm |
| Present                           | 0.229 mm | 0.183 mm | 0.137 mm |

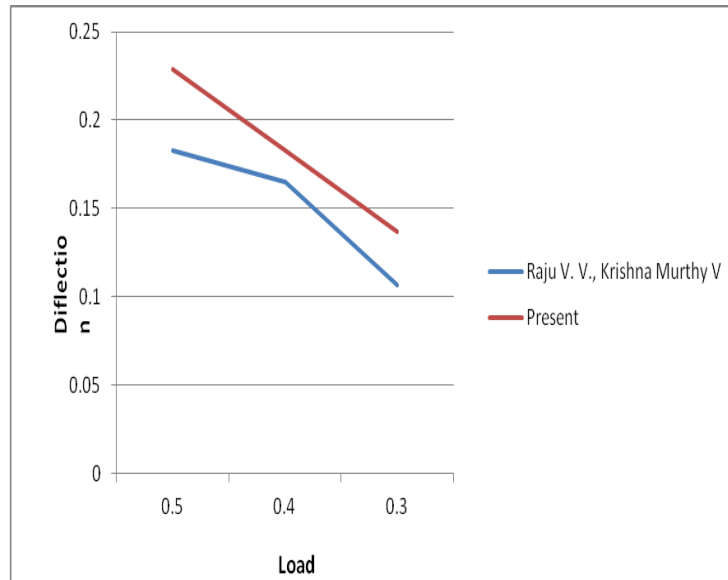


Figure: - 4. Comparison of Maximum Deflection for Cross-ply symmetric laminate

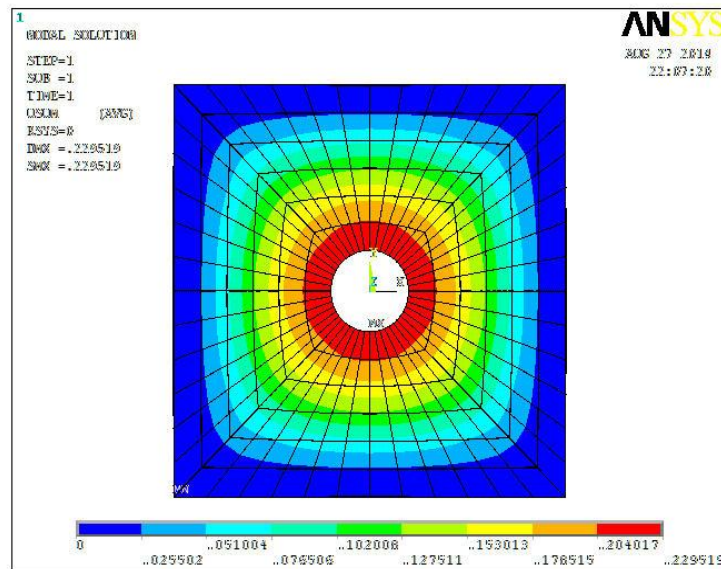


Figure: - 5. Maximum Deflection for cross-ply symmetric laminate

### 3. Vibration Analysis of Laminated Composite Plate without Cutout:-

An antisymmetric cross-ply laminate, with lamination scheme  $[0^0/90^0/0^0/90^0]$  and simply supported on all the four edges is considered. The plate is 1000 mm X 1000 mm. The laminate has side to thickness ratio of 100.

The material properties of each lamina are given in Table 3.

| Material Constant | Value     |
|-------------------|-----------|
| $E_1$             | 13.0x1010 |
| $E_2$             | 0.6 x E1  |
| $E_3$             | 0.6 x E2  |
| $\nu_{12}$        | 0.25      |
| $\nu_{23}$        | 0.25      |
| $\nu_{13}$        | 0.25      |
| $G_{12}$          | 0.6 x E2  |
| $G_{23}$          | 0.5 x E2  |
| $G_{13}$          | 0.6 x E2  |
| $\rho(kg/m^3)$    | 1500      |

First four nondimensionalized frequencies are obtained and compared with the finite element results of Kumar and Shrivastava [12] and the exact solution by Khdeir [13]. The comparison of results in Table 4 shows that the present results are in close agreement with those in [12] and [13].

#### IV. RESULT

Results are obtained in terms of nondimensionalised natural frequencies.

$$w_i = w_r S^2 / h \sqrt{(\rho/E_2)}$$

Where,

$w_i$  = Nondimensionalised natural frequency

$w_r$  = Natural frequency in rad/sec

$S$  = Length of plate in mm

$h$  = Thickness of plate in mm

$\rho$  = Density of plate in  $kg/mm^3$

$E_2$  = Young's modulus in  $N/mm^2$

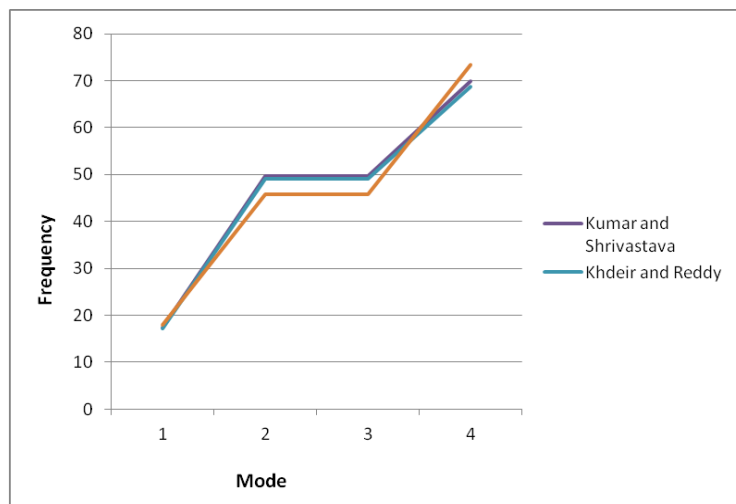


Figure: - 6. Comparison of nondimensionalized frequencies for Cross-ply antisymmetric laminate

*The results comparison as Table 4.*

| Mode | Kumar and Shrivastava [12] | Khdeir [13] | Present |
|------|----------------------------|-------------|---------|
| 1    | 17.41                      | 17.27       | 17.93   |
| 2    | 49.74                      | 49.12       | 45.72   |
| 3    | 49.74                      | 49.12       | 45.72   |
| 4    | 69.92                      | 68.74       | 73.47   |

## V. CONCLUSION

Significance of static and free vibration analysis is provided comparison of the FEA result from existing refinances of the composite laminate of different materials. This paper described static and free vibration analysis of orthotropic and procedure of laminated composite plate. These papers show the validity and accuracy of present computational method. Computer modeling is done by using commercial software ANSYS.

## VI. REFERENCES

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